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NON-LINEAR WAVES IN SLIGHTLY ANISOTROPIC ELASTIC MEDIA*

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Slightly non-linear plane waves in elastic materials possessing an arbitrary kind of intrinsic anisotropy as well as anisotropy caused by homogeneous initial deformation are considered. The influence of the deformation anisotropy on the behaviour of simple and shock waves was investigated in detail in previous papers /1-3/.

When considering low-amplitude waves the internal energy of the medium can be expanded in series in the deformations, while being constrained to terms not higher than the fourth degree. If the anisotropy influences just the form of the quadratic terms, then the internal energy can always be reduced to the same form as for deformation anisotropy with conservation of all the non-linear wave properties studied in /1-3/. If the anisotropy results in the appearance of cubic terms in the internal energy expansion that have the same order of magnitude as the quadratic terms associated with the anisotropy, then it is shown that conversion of the coordinate system in displacement gradient space can formally reduce the intrinsic anisotropy of the material for certain kinds of anisotropy, particularly, for transversally-isotropic and orthotropic media, to the same form as for anisotropy produced by preliminary elastic deformation in an isotropic material.

1. Description of the medium. The elastic medium is given by its elastic potential $\Phi = \rho_0 U(\varepsilon_{ij}, g_{ij}, d_{lmn}^{(k)} \ldots, S)$. Here U is the internal energy and S is the entropy per unit mass, ε_{ij} is Green's finite strain tensor, ρ_0 is the density of the medium in the unstressed state, g_{ij} is the metric tensor of the unstrained state, and $d_{lmn\ldots}^{(k)}$ are tensors giving the difference of the medium from an isotropic medium, for instance, the tensors giving the symmetry group.

When studying plane waves it is convenient to introduce the notation $\partial w_i/\partial x = u_i(x, t)$ where x is the wave propagation direction and w_i are the displacement vector components. The coordinates $x_1, x_2, x_3 = x$ are Lagrangian, and in the undeformed state rectangular Cartesian $(g_{ij} = \delta_{ij})$. The equations of motion for plane waves and their corresponding conditions on a discontinuity have the form /1, 4/

$$\rho_0 \frac{\partial^2 w_i}{\partial t^2} = \frac{\partial}{\partial x} \frac{\partial \Phi}{\partial u_i}, \quad \left[\frac{\partial \Phi}{\partial u_i}\right] = \rho_0 W^2 [u_i], \quad i = 1, 2, 3 \tag{1.1}$$

Here $W = dx/dt$ is the Lagrange velocity of the discontinuity. Both the initial deformations u_j^0, $\varepsilon_{\alpha\beta}$ $(\alpha, \beta = 1, 2)$ and their changes during the passage of the wave Δu_i will be considered to be small, not exceeding a certain quantity ε and we can use the expansion of the function Φ in series in ε_{ij} while being constrained to the minimum number of terms that will cover the principal non-linear effects. It is known from /1-4/ that it is necessary to expand Φ to the power ε^4 to obtain non-linear effects in transverse waves, and to the power ε^3 for longitudinal waves.

We will assume the difference of the material from isotropic to be small and we will characterize it by a certain parameter δ such that $\delta \ll \varepsilon$. In particular, the deformation anisotropy produced by the preliminary deformation $\varepsilon_{\alpha\beta} \sim \varepsilon$ $(\alpha, \beta = 1, 2)$ automatically satisfies this. The waves become quasilongitudinal and quasitransverse for a slight anisotropy of any nature and all three characteristic velocities will be distinct.

We will represent the potential Φ in the form of two components $\Phi = \Phi_0 + \Phi_1$. The first component gives an isotropic medium without initial deformations. The small second term describes the deviation of the material from isotropic. Both functions Φ_0 and Φ_1 are represented by an expansion in series in u_i (we later utilize the notation $u_1 = u$, $u_2 = v$, $u_3 = w$)

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$$\Phi_0 = {}^1/_2\lambda I_1^2 + \mu I_2 + \beta I_1 I_2 + \gamma I_3 + \nu I_1^3 + \xi I_2^2 + \ldots + \rho_0 T_0 (S - S_0) \equiv {}^1/_2\mu (u^2 + v^2) + {}^1/_2 (\lambda + 2\mu) w^2 + bw (u^2 + v^2) + aw^3 + {}^1/_4 h (u^2 + v^2)^2 + {}^1/_4 kw^4 + {}^1/_2 mw^2 (u^2 + v^2) + \rho_0 T_0 (S - S_0) \tag{1.2}$$

$$I_1 = \varepsilon_{ii},\ I_2 = \varepsilon_{ij}\varepsilon_{ij},\ I_3 = \varepsilon_{ij}\varepsilon_{jk}\varepsilon_{ki}$$

$$a = {}^1/_2\lambda + \mu + \beta + \gamma + \nu,\ 2b = \lambda + 2\mu + \beta + {}^3/_2\gamma,$$

$$h = {}^1/_2\lambda + \mu + \beta + {}^3/_2\gamma + \xi$$

$$\Phi_1 = B_1 u^2 + B_2 v^2 + B_3 w^2 + B_4 uv + B_5 uw + B_6 vw + A_1 u^3 + A_2 v^3 + A_3 w^3 + A_4 u^2 w + A_5 v^2 w + A_6 uw^2 + A_7 vw^2 + A_8 u^2 v + A_9 uv^2 + A_{10} uvw \tag{1.3}$$

Here $\lambda, \mu, \beta, \gamma, \nu, \xi$ are the elastic moduli of the medium, a, b, h, k, m are their combinations, A_i, B_i are constants describing the anisotropy, i.e., containing the tensor components $d_{ijl\ldots}^{(k)}$ and magnitudes of the initial deformations $\varepsilon_{\alpha\beta}$. We will identify the largest of the coefficients A_i, B_i by δ. Consequently, A_i, B_i of the order of ε or less, which enabled the function Φ_1 to be taken in the form of a monomial of not more than the third degree in general form which corresponds to the most arbitrary kind of anisotropy. In the absence of initial deformations A_i, B_i are constants describing the difference between the elastic properties of the medium and the isotropic properties. The function Φ does not contain linear terms, which denotes no stresses in the undeformed state.

In the general case it can be assumed that all the constants A_i, B_i are of the same order of smallness δ. But then terms with B_i in the expansion Φ_1 will be of an order greater than the terms with A_i and cubic terms cannot be written in Φ_1. In this case, in order for terms account of the anisotropy to be of the same order as the term $h (u^2 + v^2)^2$ containing the non-linearity, it is necessary to select ε such that $B_i \sim \delta \sim \varepsilon^2$. Then the form of the function $\Phi = \Phi_0 + \Phi_1$ will differ in no way from the elastic potential Φ of the isotropic medium in which the anisotropy is induced only by the term $\varepsilon_{22} - \varepsilon_{11}$ the preliminary strain. The non-linear waves (simple, shock, and the selfsimilar problem using them) are studied in detail in /1-3, 5/, and all these results then go over to the initially anisotropic media.

Moreover, we will not generally assume that A_i and B_i are of the same order. When the cubic terms will be taken into account it will be considered that the greatest coefficient A_i is much greater than all the B_i.

2. Quasilongitudinal waves. In slightly anisotropic media one quasilongitudinal wave ($\Delta w \gg \Delta u, \Delta v$) and two quasitransverse waves ($\Delta w \ll (\Delta u^2 + \Delta v^2)^{1/2}$) exist. To clarify the non-linear effects in the quasilongitudinal wave behaviour, it is sufficient to have the expansion Φ up to terms ε^3 /1, 4/, which means taking only terms with B_i in the component Φ_1. It can be seen that anisotropy of the medium of any kind for the quasilongitudinal wave will automatically be reduced to deformation anisotropy. For this, it is sufficient to transfer the origin in the space u_i, i.e., to set $u_* = u + B_5/(2b)$, $v_* = v + B_6/(2b)$, $w_* = w + B_3/(3a)$ and the medium will behave in the new variables as isotropic with preliminary elastic deformation in the quasilongitudinal wave.

3. Quasitransverse waves. By using (1.1) for quasitransverse waves the longitudinal component w can be eliminated by expressing it in teams of u and v. Consequently, the two-dimensional potential $F(u_\alpha)$ ($\alpha = 1, 2$) can be introduced in place of the elastic potential $\Phi(u_i)$ ($i = 1, 2, 3$). The procedure for eliminating $u_3 = w$ is described in detail in /6/. The initial assumptions of this paper that the second derivatives of Φ_1 with respect to u, v and w, u or w, v do not exceed, respectively $\max\{\varepsilon^2, \delta\}$ and $\max\{\varepsilon, \delta\}$ where $\varepsilon = \max\{u, v\}$ are satisfied. The system of Eqs.(1.1) is simplified here and contains two equations of motion or two corresponding relationships on the discontinuity

$$\rho_0 \frac{\partial^2 w_\alpha}{\partial t^2} = \frac{\partial}{\partial x}\frac{\partial F}{\partial u_\alpha},\quad \left[\frac{\partial F}{\partial u_\alpha}\right] = \rho_0 W^2 [u_\alpha],\quad \alpha = 1, 2 \tag{3.1}$$

The expression

$$F = {}^1/_2 (f - g) u^2 + {}^1/_2 (f + g) v^2 - {}^1/_8 \varkappa (u^2 + v^2)^2 \tag{3.2}$$

was obtained for F in /6/ in which a quadratic representation was taken for Φ_1 (cubic terms in (1.3) were not taken into account), where $f, g, \varkappa$ are constant coefficients. The small quantity g is the sole parameter taking complete account of the anisotropy under the assumption that Φ_1 contains only quadratic terms. The elastic constant $\varkappa$ characterizes the non-linear properties of the medium. The qualitative difference in the non-linear effects depends on the sign of $\varkappa$. For an isotropic medium with the initial strains $\varepsilon_{11}, \varepsilon_{22}$ (the x_1, x_2 axes in the plane of the wavefront are selected so that $\varepsilon_{12} = 0$)

$$f = \mu + 2bI_1^0 - (\mu + {}^3/_4\gamma)(\varepsilon_{11} + \varepsilon_{22}),\quad g = (\mu + {}^3/_4\gamma)(\varepsilon_{22} - \varepsilon_{11})$$
$$\varkappa = \mu + (\mu + \beta + {}^3/_2\gamma)^2/(\lambda + \mu) - 2\xi = 4b^2/(\lambda + \mu) - 2h$$

If there is no initial strain in the isotropic medium, then $f = \mu,\ g = 0$ /4/.

For a medium with anisotropy of the general form (1.3) (taking cubic terms into account) we have for the arbitrary axes x_α

$$\begin{aligned} F &= {}^1/_2\,(f - g)u^2 + {}^1/_2\,(f + g)v^2 - {}^1/_8\varkappa\,(u^2 + v^2)^2 + suv + \\ &\quad {}^1/_2\,(pv + qu)(u^2 + v^2) + eu^3 + dv^3 \\ f &= \mu + B_1 + B_2 - \frac{B_5^2 + B_6^2}{2(\lambda + \mu)},\quad g = B_2 - B_1 + \frac{B_5^2 - B_6^2}{2(\lambda + \mu)} \\ p &= A_6 - \frac{bB_6}{\lambda + \mu},\quad q = A_9 - \frac{bB_5}{\lambda + \mu},\quad s = B_4 - \frac{B_5B_6}{\lambda + \mu} \\ d &= A_2 - A_8,\ e = A_1 - A_9 \end{aligned} \tag{3.3}$$

Since there is a sufficiently complete investigation of the non-linear elastic waves /1-3, 5/ for the case of deformation anisotropy (3.2), it is reasonable to clarify when and how the function F reduces to the form (3.2) for the general case of the anisotropy (3.3). It turns out that this is possible only for $e = d = 0$, i.e., for media with certain symmetry properties when $A_1 = A_9,\ A_2 = A_8$ in (1.3). It will be shown below that precisely such a property is possessed by transversally-isotropic and orthotropic elastic media.

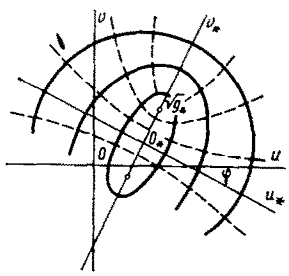

Fig.1

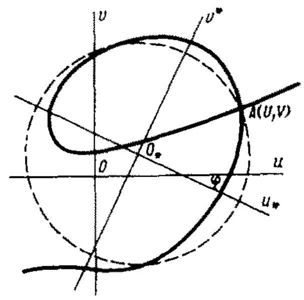

Fig.2

For $e = d = 0$ the cubic terms in expression (3.3) for F can be eliminated by a parallel transfer of the coordinate axes in the uv plane. The new origin O_* should be at the point $(2q/\varkappa,\ 2p/\varkappa)$. By a subsequent rotation of the axes around the new origin O_* through an angle φ the function F for the anisotropic medium is reduced to a form that agrees with (3.2) (the linear terms are denoted by the multiple dots)

$$\begin{aligned} F &= {}^1/_2(f_* - g_*)u_*^2 + {}^1/_2(f_* + g_*)v_*^2 - {}^1/_8\varkappa(u_*^2 + v_*^2)^2 + \ldots \\ f_* &= f + 4\frac{p^2 + q^2}{\varkappa},\quad g_* = \left[\left(g + 2\frac{p^2 - q^2}{\varkappa}\right)^2 + \left(s + 4\frac{pq}{\varkappa}\right)^2\right]^{1/2} \\ u_* &= (u - 2q/\varkappa)\cos\varphi + (v - 2p/\varkappa)\sin\varphi \\ v_* &= (-u + 2q/\varkappa)\sin\varphi + (v - 2p/\varkappa)\cos\varphi \\ \operatorname{tg} 2\varphi &= -(s + 4pq/\varkappa)/[g + 2(p^2 - q^2)/\varkappa] \end{aligned} \tag{3.4}$$

The stresses in the new coordinate system are non-zero in the undeformed state, as is indicated by the appearance of the linear terms in the function F because of the coordinate transformation. However, neither the shock adiabatic equation nor the equation of the plane wave integral curves and the expression for the characteristic velocities contain these terms. Rotation of the u, v axes corresponds to rotation of the x_1, x_2 axes in physical space. Up to now these axes have been arbitrary in the plane of the wave front and their selection can be managed. The description of the non-linear elastic wave behaviour in an anisotropic medium in the new variables u_*, v_* is the same as in a preliminary deformation anisotropic medium. All the results obtained in /1-3, 5/ remain valid.

Integral curves of quasitransverse simple waves are presented in Fig.1 and the shock adiabatic for the shocks and a circle on which the entropy is $S = \text{const}$ are shown in Fig.2. The radius of the entropy circle equals $R = [(U - 2q/\varkappa)^2 + (V - 2p/\varkappa)^2]^{1/2}$ where U, V are the values of u, v ahead of the discontinuity.

4. Transversally-anisotropic and orthotropic media. These media are used in many problems of mechanics as models of anisotropic materials, where the difference of a medium from isotropic will often be slight. For instance, the anisotropy of materials, caused by their fabrication (stamping and rolling), and the anisotropy of rocks comprising the upper mantle of the earth, are small and possess certain symmetry properties.

For an isotropic medium its geometry is described by the metric tensor g_{ij}. To describe the deviation of a medium from isotropic, the tensors $d_{ij...}^{(k)}$ are introduced. There is one such tensor in the medium under consideration and its special form can be indicated due to the symmetry properties. There is a certain derived direction $\mathbf{l}$ in planes orthogonal to it in a transversally-isotropic medium (TIM) and the properties of the medium are isotropic. Since the direction of this axis is immaterial, it is given by the square of the vector $\mathbf{l}$, i.e., the tensor $l_{ij} = \alpha_i\alpha_j$ where α_i are proportional to the direction cosines of the axis $\mathbf{l}$ /7/. Orthotropic media possess three mutually perpendicular planes of symmetry, which can be given by a symmetric tensor of the second rank d_{ij} /7/. The TIM model is a special case of an orthotropic medium when $d_{ij} = \alpha_i\alpha_j$ for the mathematical description.

Thus an elastic medium is given by its potential $\Phi = \Phi(\varepsilon_{ij}, g_{ij}, d_{ij}, S)$. Only six independent scalar invariants containing the strain components I_1, I_2, I_3 presented in (1.2) and $K_1 = d_{ij}\varepsilon_{ij}$, $K_2 = d_{ij}\varepsilon_{jk}\varepsilon_{ki}$, $K_3 = d_{ij}d_{jk}\varepsilon_{ki}$ can be composed from the three second-rank tensors $\varepsilon_{ij}, g_{ij}, d_{ij}$. Therefore $\Phi = \Phi(I_1, I_2, I_3, K_1, K_2, K_3, S)$. Note that there are only five independent invariants in TIM, since the invariant K_3 is proprotional to K_1.

We will assume that the deviation of a medium from isotropic is small. For a quantitative description of this fact we assume that the tensor components d_{ij} have an order of smallness $\delta < \varepsilon$. This is not the only method of describing the smallness of the anisotropy but we will use it. Then the invariants K_i are obviously of the order of $K_1 \sim \varepsilon\delta$, $K_2 \sim \varepsilon^2\delta$, $K_3 \sim \varepsilon\delta^2$.

Following the previous method, we will use an expansion for the part of the elastic potential Φ_1 taking account of the anisotropy in which terms up to a total of the fourth order of smallness in ε and δ are written down

$$\Phi_1 = a_1K_1^2 + a_2K_2 + a_3K_1I_1 + a_4K_1I_2 + a_5K_1I_1^2 + a_6I_1K_2 + a_7K_3I_1$$

The coefficients a_i are the elastic constants of the medium: the magnitudes are finite. For TIM $a_7 = 0$.

Changing to the variables u, v, w we find expressions for the coefficients A_k, B_k in (1.3) in terms of the components of the tensor d_{ij} and $\varepsilon_{\alpha\beta}$, $\alpha, \beta = 1, 2$. As is seen from (1.3), the calculation for A_k should here be performed to an accuracy ε and for B_k to an accuracy ε^2. In quasitransverse waves $w \sim \varepsilon^2$, consequently, the terms with $A_3, A_4, \ldots, A_7, A_{10}, B_8$ take no part in the consideration, while the coefficients B_5 and B_6 should be calculated to accuracy ε. Firstly, we obtain

$$A_1 = A_9 = {}^1/_2 a_4 d_{13},\quad A_2 = A_8 = {}^1/_2 a_4 d_{23}$$

i.e., for TIM and orthotropic media the coefficients e and d discussed in Sect.3 above vanish. This means that for these materials the investigation of the non-linear elastic waves will result in the case of deformation anisotropy /1-3, 5/, as has been shown in Sect.3.

The two-dimensional elastic potential F has the form (3.3) while the coefficients are calculated in terms of the components of d_{ij} from the formulas

$$\begin{gathered}
p = {}^1/_2\omega d_{23},\quad q = {}^1/_2\omega d_{13},\quad \omega = a_4 - 2b\,(a_2 + a_3)/(\lambda + \mu)\\
s = 2\,(\mu + {}^3/_4\gamma)\varepsilon_{12} + {}^1/_2 d_{12}m - d_{13}d_{23}\,(a_2 + a_3)^2/(\lambda + \mu)\\
g = (\mu + {}^3/_4\gamma)(\varepsilon_{22} - \varepsilon_{11}) + {}^1/_4 m\,(d_{22} - d_{11})\\
f = \mu + 2bI_1^\circ - (\mu + {}^3/_4\gamma)(\varepsilon_{11} + \varepsilon_{22}) + 2\xi\varepsilon_{12}^2 - {}^1/_2 a_1 d_{33} +\\
{}^1/_4 m\,(d_{11} + d_{22} + 2d_{33}) + {}^1/_2 a_4\,(\varepsilon_{11}d_{11} + \varepsilon_{22}d_{22} + 2\varepsilon_{12}d_{12})\\
m = a_2 + 4a_1d_{33} + a\,(\varepsilon_{11} + \varepsilon_{22})
\end{gathered}$$

In order to reduce the function F to the form (3.2), the origin in the uv plane must be shifted to the point $O_*(\omega d_{13}, \omega d_{23})$. The expression for $s + 4pq/\varkappa$ in formula (3.4) to evaluate the angle φ contains components of the initial strain and the tensor d_{ij}.

From the very beginning, axes x_1, x_2 can be selected in the formulation of the problem in which these components are connected by the relationship

$$2\,(\mu + {}^3/_4\gamma)\varepsilon_{12} + {}^1/_2 m d_{12} - d_{13}d_{23}\,[(a_2 + a_3)^2/(\lambda + \mu) + \omega^2/\varkappa] = 0$$

In such axes the function F again has the form (3.2) and the anisotropy parameter is $g_* = g + {}^1/_2\omega^2\,(d_{23}^2 - d_{13}^2)\varkappa$.

In particular, $d_{ij} = \alpha_i\alpha_j$ for TIM, where α_i are proportional to the direction cosines of the axis $\mathbf{l}$ and have a magnitude $\sim\delta^{1/2}$. Then

$$\begin{gathered}
p = {}^1/_2\omega\alpha_2\alpha_3,\quad q = {}^1/_2\omega\alpha_1\alpha_3,\quad g = (\mu + {}^3/_4\gamma)(\varepsilon_{22} - \varepsilon_{11}) +\\
{}^1/_4 m\,(\alpha_2^2 - \alpha_1^2)\\
s = 2\,(\mu + {}^3/_4\gamma)\varepsilon_{12} + \alpha_1\alpha_2\,[{}^1/_2 m - \alpha_3^2\,(a_2 + a_3)^2/(\lambda + \mu)]
\end{gathered}$$

If there is no deformation anisotropy, i.e., $\varepsilon_{\alpha\beta} = 0$ then the coordinate axes x_1, x_2 should be selected so that one of them (x_2, say) coincides with the projection of $\mathbf{l}$ on the plane of the wave front. Then $\alpha_1 = 0, s = 0,\ g_* = {}^1/_4 m\alpha_2{}^2$. As is shown in /1, 2/, in order to be able to describe the behaviour of shocks in the whole uv plane, i.e., to have the complete shock adiabatic passing through the point $A\ (U, V)$ corresponding to the state before the jump (Fig.2), the anisotropy parameter g_* should be small, of the order of $R^2 \sim \varepsilon^2$, where R is the radius of the circle passing through the origin on which $S =$ const. In this case $R^2 = U^2 + (V - \omega\alpha_2\alpha_3)^2$. In order that $g_* \sim R^2$, it is either necessary to have a sufficiently small anisotropy such that $\alpha_2 m^{1/2} \sim \varepsilon$ (at least along the x_2 axis), or the quantity α_2 is small because the direction of wave propagation (the x axis) is close to $\mathbf{l}$.

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TWO APPROACHES TO THE INVESTIGATION OF ANTIPLANE DEFORMATION OF AN ISOTROPIC SOLID WITH A THIN ELASTIC INCLUSION*

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An approach is proposed to the investigation of the state of stress and strain of a piecewise-homogeneous plane consisting of a matrix and a thin tunnel-like rectangular inclusion with rounded-off corners under the assumption that such a composite body is under antiplane deformation conditions. A numerical comparison is made of the results obtained in this paper and on the basis of an approximate model /1/. It is shown that they agree satisfactorily at sufficiently large distances from the vertices of the inclusion, when the inclusion is more pliable than the matrix.

1. We assume that ideal mechanical contact conditions are satisfied on the material interfacial line L. We select an $OxyZ$ system of Cartesian coordinates with origin at the centre of a rectangular inclusion and the OZ axis directed along the axis of body deformation. We know that the function reflecting the unit circle γ on the contour L has the form /2, 3/

$$z = x + iy = \omega(\sigma) = R\left(\sigma + \sum_{k=1}^{n} c_k \sigma^{-k}\right), \quad \sigma \in \gamma \tag{1.1}$$

$$R = \left(1 + \sum_{k=1}^{n} c_k\right)^{-1}$$

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